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Fine Reconstruction of Seismic Data Using Localized Fractals

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Abstract

Seismology requires accurate (fine) data reconstruction from sparsely (or irregularly) sampled data sets, but such results are usually not possible with conventional (non-fractal) methods. To produce a high-precision reconstruction of seismic data, a more accurate localized fractal reconstruction approach can be used provided the data is self-similar on local and global spatial scales. In this paper, a novel localized fractal reconstruction approach has been presented. This method is a data-driven algorithm that does not require any geological or geophysical assumptions concerning the data. Here, we report our results of using the approach to reconstruct sparsely sampled seismic data. Our results indicate that the fine structure associated with seismic data can be easily and accurately reconstructed using the localized fractal approach, indicating that seismic data is indeed self-similar on local and global spatial scales. This result holds promise not only for future seismic studies, but also for any field that requires fine reconstruction from sparsely sampled data sets.

Keywords: Fine reconstruction of seismic data, localized fractals, high-precision interpolation, fine structure

1. Introduction

Like many research fields in geophysics, seismology depends heavily on observational data. Seismic data are typically irregularly (or sparsely) sampled along spatial coordinates. This is often caused by the presence of obstacles, no-permit areas, feathering, and dead traces and by economics. Most multitrace data processing algorithms cannot adequately handle irregular sampling. This can make it difficult to obtain detailed information on subterranean structures or to avoid spatial aliasing. Numerical reconstruction of seismic data is therefore a necessity. In particular, interpolating seismic traces is often used for seismic data reconstruction, but unfortunately, conventional interpolation methods typically fail to provide the level of detail needed to understand fine structures present in seismic wavefields. Novel reconstruction approaches that would provide both high-precision and fine interpolation of seismic traces would be advantageous.

High-precision interpolation can present a problem for any data-driven research field. In seismology, conventional interpolation approaches (Hindriks and Duijndam, 2000; Duijndam and Schonewille, 1999; Kabir and Verschuur, 1995; Larner and Rothman, 1981; Liu, 2004; Ronen, 1987; Spitz, 1991; Trad et al., 2003; Porsani, 1999; Wang, 2003; Zwartjes and Sacchi, 2007) are valid for de-aliasing of seismic events, but in general seismic data are self-similar on both local and global scales. Standard interpolation approaches are not the most optimal method for dealing with data that

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exhibit local-global self-similarity. Instead, reconstructing data using fractal methods may yield superior results, since fractals are themselves self-similar on local and global spatial scales. Overall, fractal interpolation functions are powerful tools that can reconstruct the local properties of complex data sets. In comparison, non-fractal interpolation functions are easily adapted to smooth data sets, but typically ignore the fine structures and local properties of more complicated data sets.

Conventional fractal interpolation methods based on the Random Fractional Brown Motion model (Navascués and Sebastián, 2004; Barnsley and Demko, 1985; Fan, 2005) assign the same value for the vertical scaling factor throughout the entire interpolated region. This makes it difficult to obtain information on the local properties of the data. In addition, conventional fractal interpolation methods do not have any explicit expression, so the approach is often a complicated, iterative process. Despite these challenges, there is recent progress in using localized and explicit fractal interpolation functions to obtain detailed information on the local properties of seismic data sets.

In this paper, an approach for fine reconstruction of seismic data approach has been proposed that does not require any geological or geophysical assumptions concerning the data. The reconstruction approach explored is based on a localized fractal interpolation function. Using the approach, the fine structure associated with seismic data can be easily and accurately reconstructed. To demonstrate the reliability and the validity of the approach, numerical examples for high-precision interpolation and fine reconstruction of real seismic data have been given in this paper.

2. Theory and method

The new fractal reconstruction method described below is derived under the assumption that the data is self-similar on local and global spatial scales. This method is a data-driven algorithm that does not require any geological or geophysical assumptions concerning the data.

We use a localized fractal interpolation to reconstruct seismic data and associated fine structures. To increase the method’s efficiency, we also introduce the inorder traversing binary tree into the reconstruction.

The localized fractal interpolation approach stated previously is based on the concepts of affine transform and iterated function systems [Singer and Zajdler, 1999; Navascués and Sebastián, 2004; Barnsley and Demko, 1985; Chu and Chen, 2003; Fan, 2005; Sun et al., 1996]. Generally, the affine transform is defined by

$$\omega_n\begin{pmatrix} x \\ f(x) \end{pmatrix} = \begin{pmatrix} L_n(x) \\ F_n(x,y) \end{pmatrix}, \quad n \in \{1, 2, \cdots, N\} , \tag{1}$$

where $n = 1, 2, \cdots, N$. In the following, we localize the affine transform and present a localized fractal interpolation approach.

When $L_n(x)$ and $F_n(x, y)$ are assumed to be linear functions and mapping from Equation (1) is localized, t hen the mapping can be defined as the affine transform

$$\omega_n\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_n & 0 \\ c_n & d_n \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \end{pmatrix} \quad n = 1, 2, \cdots, N , \tag{2}$$

where coefficients a_n , c_n , e_n and f_n are determined by conditions $\omega_n(x_0, y_0) = (x_{n-1}, y_{n-1})$ and $\omega_n(x_N, y_N) = (x_n, y_n)$. Here we have d_n ($|d_n| < 1$) as a free parameter, which is called the vertical scaling factor and plays a key role during the interpolation. Assuming fractal interpolation, the localized and explicit fractal interpolation function can be given by

$$f(x) = c_n x + d_n f(x) + f_n , \tag{3}$$

or

$$f(x) = \frac{c_n x + f_n}{1 - d_n}, x \in [x_{n-1}, x_n] . \tag{4}$$

The vertical scaling factor d_n can be expressed as

$$d_n = \frac{y_n - y_{n-1}}{\varepsilon \cdot \sqrt{(y_{\max} - y_{\min})^2 + (y_n - y_{n-1})^2}}, \qquad (5)$$

where $y_{\max}$ and $y_{\min}$ are chosen over the interval $[x_{n-l}, x_{n+l}]$, and ε is determined by $\varepsilon = 1.0 + random(n)$. $random(n)$ is a random deviate generated from a uniform (0, 1) distribution. Because $random(n)$ is unique at the *nth* point, the vertical scaling factor d_n is unique. Therefore the value of interpolation function is unique. Equations (4) and (5) are key formulas for the localized fractal interpolation approach.

3. Numerical examples on high-precision interpolation and fine reconstruction of real seismic data

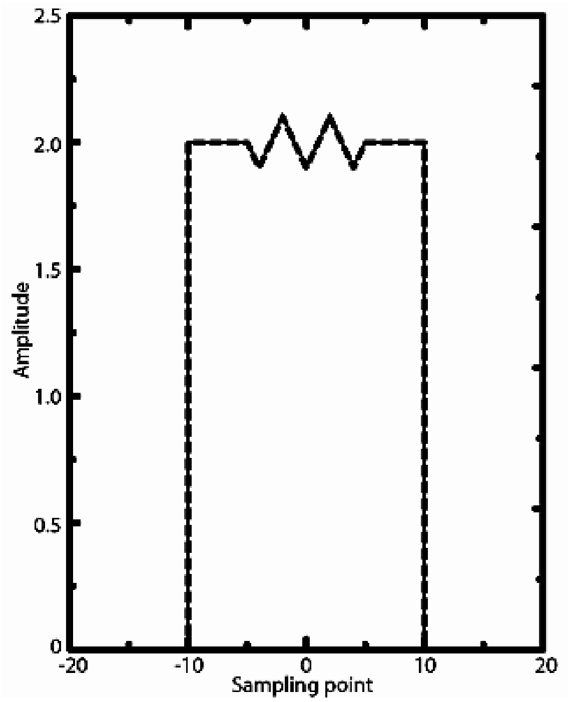
To testify the reliability and the validity of the localized fractal interpolation, we carry out a typical reconstruction example by the interpolation for a step-triangle curve and Ricker wavelet. Figure 1A, B show the original curves and fitted curves for step-triangle curve and Ricker wavelet, respectively. The dotted lines are obtained by using the interpolation and the solid lines are the exact curves. Although the new fractal interpolation approach appears simple, it is also precise (see Figure 1) and efficient. It is suitable for treating both unsmoothed (Figure 1A) and smoothed (Figure 1B) data structures, and faithfully reconstructs the local properties of a data set (Figure 1A).

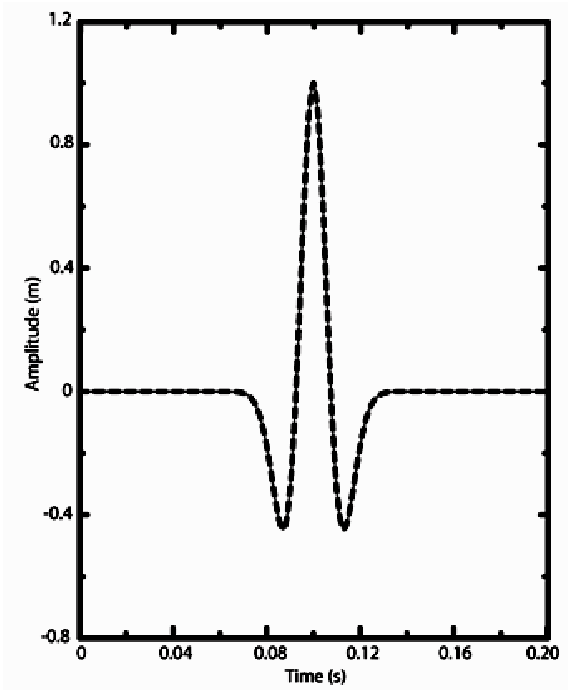
We use the localized and explicit fractal interpolation function to reconstruct high-resolution seismic data from sparse seismic traces. The example considered here uses seismic data from the Jiyang Depression and the Southern Bohai Bay Basin in Northern China, as identified in Figure 2 (*Zhao et al.*, 2004). We selected records of a seismic event at seismic stations 143SLX, 137HJZ, 131LJZS, 125XFX, 119YFZ, 113YZC, 107XZX, 101QDZ, 95QDZ and 87SZZ (Figure 2), which produced the seismogram section shown in Figure 3A. The average spacing between stations was 10 km. The parameters of seismic event 010102 are listed in Table 1. The seismic data were bandpass-filtered between 0.005 and 4 Hz and instrument responses were removed. Since the seismogram is complicated and irregular, conventional interpolation approaches will not reproduce the local properties and fine structure of the data. We erased the even traces in the seismogram section in order to construct a sparsely sampled seismogram (Figure 3B) for reconstruction. Figure 3C shows the seismogram section after interpolating with the localized and explicit fractal approach.

Table 1. Parameters for seismic event

Event	Origin (UT) (dd/mm/yy)	Latitude	Longitude	Depth (km)	M_w
010102	02/01/01; 07:30:04	126.81	6.75	33	6.4

M_w denotes the magnitude of the event.





A **B**

Fig. 1: Testing precision for the localized and explicit fractal interpolation function. **A.** Original data curve (solid line) and interpolated data curve (dotted line). **B.** Original Richer wavelet (solid line) and interpolated wavelet (dotted line).

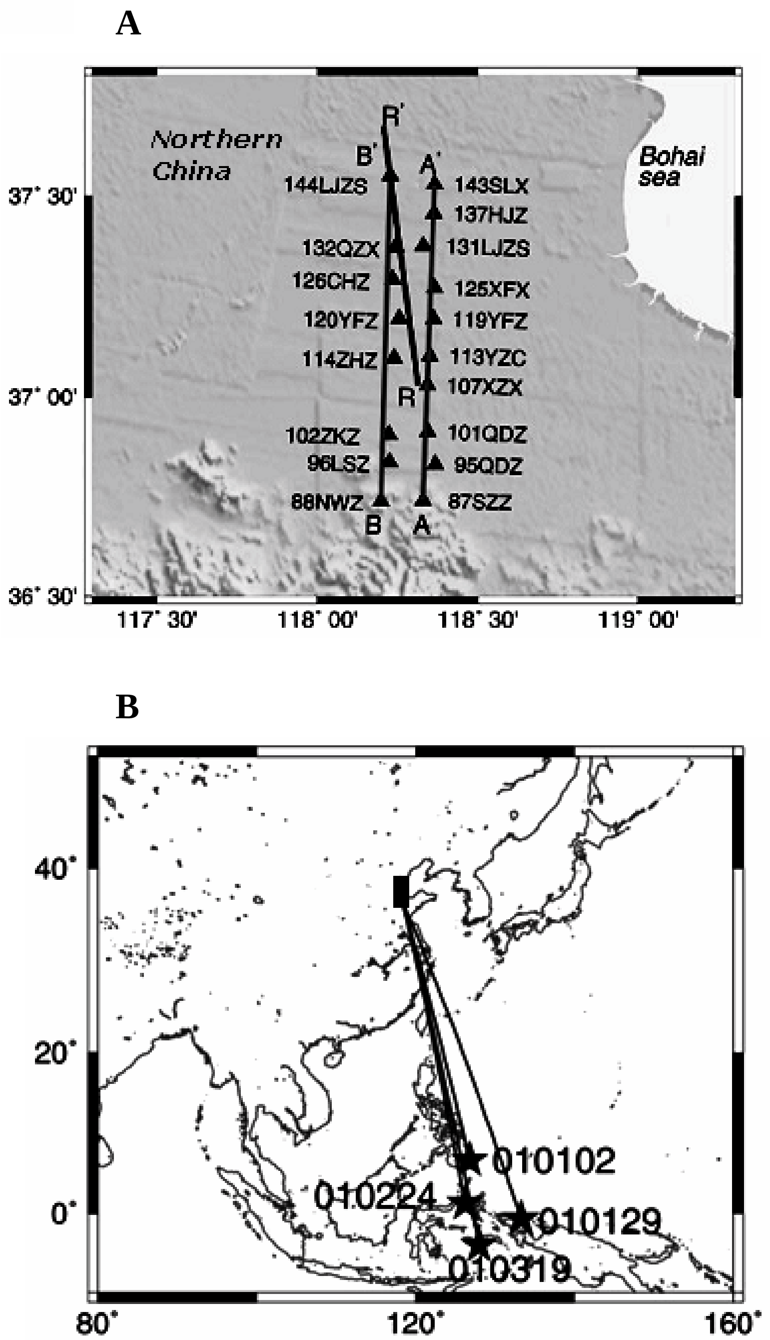


Fig. 2:. A. Location map of portable seismic stations (black triangles 143SLX, 137HJZ, 131LJZS, 125XFX, 119YFZ, 113YZC, 107XZX, 101QDZ, 95QDZ and 87SZZ) deployed in the Jiyang depression and the southern Bohai Bay Basin, Northern China, along with topography in the region. **B.** Locations of seismic events (black stars), portable seismic stations (black rectangles), and great circle paths. In this research, seismic event 010102 was chosen for study.

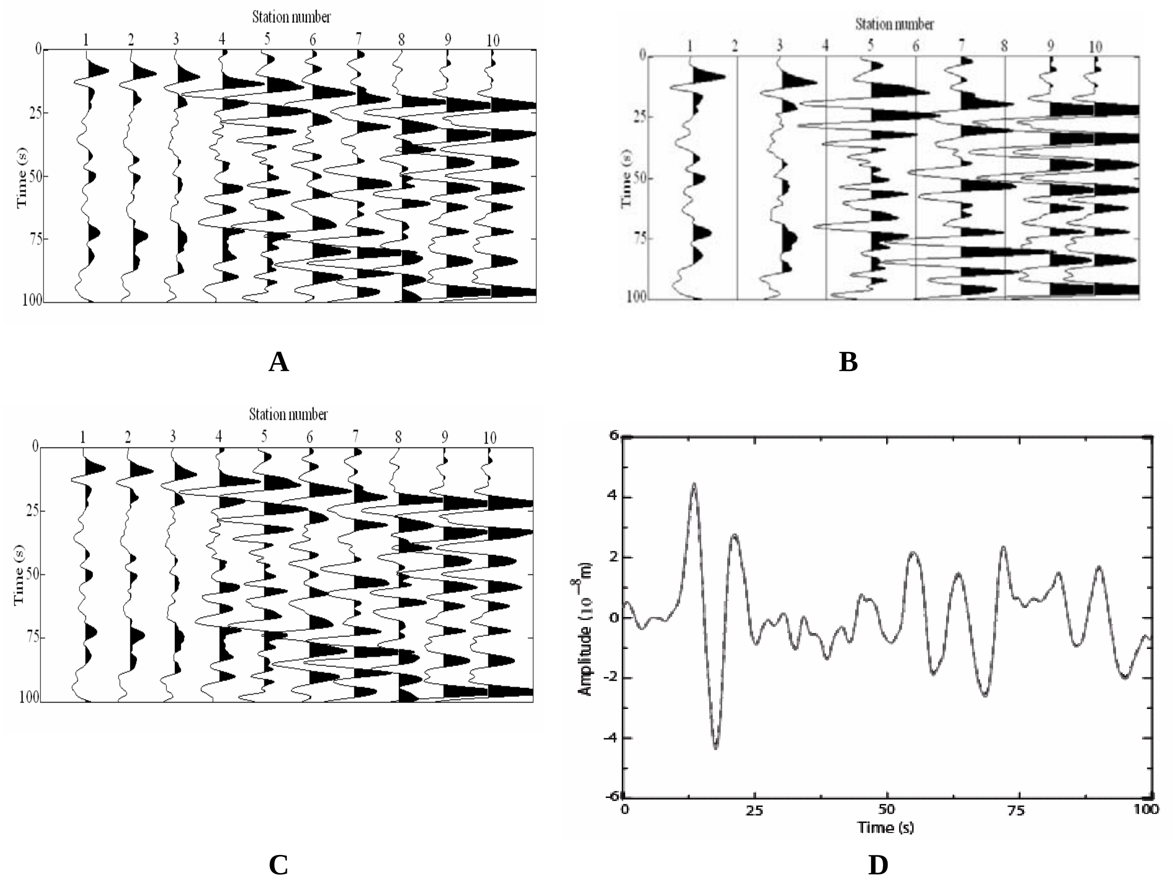


Fig. 3: Reconstruction of seismic data. **A.** Original seismogram section from seismic traces of event 010102 (Figure 2B) at seismic stations (black triangles) 143SLX, 137HJZ, 131LJZS, 125XFX, 119YFZ, 113YZC, 107XZX, 101QDZ, 95QDZ and 87SZZ (line AA in Figure 1a). **B.** Sparse seismogram section (the even traces are erased from panel a). **C.** Interpolated (reconstructed) seismogram section, including the even traces from Figure A and the interpolated traces. **D.** Comparison of original seismogram (solid line) at Station 107XZX and interpolated (reconstructed) seismogram (dotted line).

Comparing the original and interpolated (reconstructed) seismogram sections, we find that the two are essentially identical in amplitude, phase and waveform. To verify the precision and fidelity, we compared an interpolated (reconstructed) trace with its original trace (as erased in the initial section). Indeed, we find a near-perfect match (Figure 3D). By correlation analysis for the reconstructed trace and its original trace (Figure 3D), it is obtained that the coefficient of determination R^2 is 0.98. Each of these traces is provided with 4000 sampling points. These results indicate that the localized and explicit fractal interpolation can efficiently and exactly deal with the problem of complicated data structures, provided the data is self-similar on local and global spatial scales. Such a result would not be possible using conventional interpolation methods. Also, since the reconstruction is faithful to the original data, it supports the assertion that seismic data have the local-global self-similarity property associated fractal geometry. Actually, a fine reconstruction of complex data sets is realized by using the localized fractal approach in this paper. The fine reconstruction is data-driven.

4. Discussion and conclusions

In this paper, the seismicdata reconstruction approach based on a localized fractal interpolation function has been proposed that does not require any geological or geophysical assumptions concerning the data. Although this method is a data-driven algorithm that does not require any geological or geophysical assumptions concerning the data in theory, the seismic data observed contains geological or geophysical information of the studied region and is self-similar on local and global spatial scales. Therefore the seismic data reconstructed from the seismic data observed will be objective, correct and reliable. The new approach is numerically stable and easier to implement. The method is robust enough to generate near-perfect results. Also, results of using the approach to reconstruct sparsely sampled seismic data have been obtained. These results show that the use of high-precision and high-fidelity fractal interpolation scheme for data reconstruction is a valid and efficient approach not only for seismic wavefield de-aliasing, but also for fine and amplitude-preserving reconstruction of seismic data from sparse seismic traces.

Results stated previously demonstrate the usefulness of localized fractal interpolation for seismic data reconstruction, and in particular its ability to obtain detailed information for high-resolution seismic imaging and high-precision reconstruction of seismic wavefields. Indeed, these results are significant for any research field that requires accurate interpolation and fine reconstruction of sparsely sampled and irregular data sets, provided the data exhibits local and global self-similarity. Also note that our approach above only considered first-order interpolation, and it remains to be seen if incorporating higher orders will affect the results. Extending the approach to higher dimensions may yield a more precise, reliable and economical method for high-resolution imaging and high-precision wavefield reconstructions.

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