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PS-wave *Q* estimation based on the P-wave *Q* values

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Abstract

Through assumption of the equivalent velocity and equivalent quality factor of the PS-wave, in visco-elastic media, PS-wave Q estimation can be realized with the P-wave quality factor and P- to S-wave velocity ratio. For sedimentary rock, which has strong agglutination, the internal friction is mainly contributed by the shear friction along crevices or inter-granular crevices, so that a relationship between PS-wave Q values and P-wave Q values can be built up even when the S-wave Q values are unknown. In the estimation of the PS-wave quality factor, the P- to S-wave velocity ratio can be computed based on two-way traveltimes of PP and PS events respectively, in order to avoid the influence of the inaccuracy of P- and S-wave velocities. The method is demonstrated with a zero-offset VSP data in a coal-mining field. The results of P-wave and PS-wave Q values estimated show a consistency with lithology revealed from drilling.

Keywords: equivalent velocity, equivalent quality factor, PS-wave, visco-elastic, P- to S-wave velocity ratio

1. Introduction

Q compensation of the converted (PS) wave is an interesting problem in multi-component seismic data processing. In particular, the estimation of the PS-wave quality factor, under the assumption of visco-elastic and isotropic media, is important in PS-wave data processing.

Inverse Q filtering is a key step in seismic data processing (Wang 2008a). But PP-wave Q compensation studies get more attention than PS-wave Q compensation. Wang (2002, 2006) introduced stabilized inverse Q filtering approaches, such as wave field downward extrapolation and Gabor transform methods, for the depth-dependent, layered Q model and for the lateral and vertical variable Q model. In addition, Wang and Guo (2004) and Wang (2008b) also suggested to combine inverse Q filtering in a migration processing, to produce high-resolution seismic images.

For Q model estimation, besides the conventional spectral ratio method, Wang (2004) put forward Q analysing methods based on either the attenuation or compensation functions. These methods may result in a more stable result than the conventional spectral ratio method (Wang 2003). Dasios *et al* (2001) also suggested that an instantaneous frequency method is more stable than the spectral ratio method. According to White (1992), the instantaneous frequency values provide maximum stability when the amplitude envelope is large.

However, when compared to PP-waves, PS-waves show a different frequency bandwidth and different dominant frequency (Deffenbaugh 2000). Such inconsistency on frequency characters increases the difficulty of geological prediction using PP- and PS-waves jointly. In order to realize comparative interpretation and analysis on both PP- and PS-waves, the preservation of relative amplitudes is required in multi-component seismic data processing. Zhao and Wang (2004) suggested firstly to correct spherical divergence and then to make Q compensation for PP- and PS-wave reflections separately. The PS-wave Q model can also be inversed through calculation of the S-wave quality factor (Yan and Liu 2009), accompanied essentially with S-wave well logging or three-component vertical seismic profile (VSP) survey.

In this short note, an equivalent quality factor for PS-waves in the post-stack domain is derived. It is defined in terms of the PP-wave quality factor and the P- to S-wave velocity ratio. Furthermore, the velocity ratio does not need explicit P- and S-wave velocities and can be calculated based on the PP- and PS-wave two-way time in the stack sections. Therefore, the PS-wave Q compensation is possible even in the case of no information about the S-wave velocity and S-wave quality factor.

2. Theoretical background

Fundamentally, PS-wave Q compensation needs both the P-wave quality factor Q_P and S-wave quality factor Q_s, for asymmetric PS-wave ray path in visco-elastic media. For a post-stack PS seismic trace, where we can assume the source-receiver offset be zero, the PS-wave propagation can be written as (Richard and Stewart 2002)

$$A_{PS} = A_0 \exp\left[-\pi f x \left(\frac{1}{Q_P V_P} + \frac{1}{Q_S V_S}\right)\right], \tag{1}$$

where A_{PS} is the PS-wave amplitude, A_0 is the initial source amplitude, x is a half of the propagation distance (i.e. from the surface down to the reflector), f is the frequency and V_P and V_S are the velocities of P- and S-wave respectively.

Assuming there is an equivalent velocity V_{PS}, and an equivalent quality-factor Q_{PS} for the PS-waves, the PS-wave propagation can be re-written as

$$A_{PS} = A_0 \exp\left(-\frac{2\pi f x}{Q_{PS} V_{PS}}\right). \tag{2}$$

where the equivalent velocity V_{PS} is defined as

$$\frac{1}{V_{PS}} = \frac{1}{2}\left(\frac{1}{V_P} + \frac{1}{V_S}\right), \tag{3}$$

and then the equivalent quality factor can be derived as

$$\frac{1+\gamma}{Q_{PS}} = \frac{1}{Q_P} + \frac{\gamma}{Q_S}, \tag{4}$$

in which $\gamma = V_P/V_S$ is the ratio of the P-wave velocity to the S-wave velocity. This equation shows that the equivalent quality factor of the PS-wave is related to the velocity ratio and quality factor of P- and S-waves.

In the rest of this section, we derive a formula for Q_S^{-1} and in turn a formula for Q_{PS}^{-1}, expressed in terms of Q_P^{-1}.

For sedimentary rock, the P-wave quality factor Q_P and S-wave quality factor Q_S can be written as (Meissner 1986)

$$Q_P = \frac{\kappa^* + \frac{4}{3}\mu^*}{\kappa + \frac{4}{3}\mu}, \qquad Q_S = \frac{\mu^*}{\mu}, \tag{5}$$

where κ and μ are the bulk and shear modulus of the elastic component in visco-elastic media respectively, and κ^* and μ^* are the bulk and shear modulus of the viscosity component.

Since there are always the relationships as following in homogeneous and isotropic media:

$$k + \tfrac{4}{3}\mu = \rho V_P^2, \qquad \mu = \rho V_S^2, \tag{6}$$

we can obtain

$$\frac{Q_P}{Q_S} = \gamma^2 \cdot \frac{1}{\frac{\kappa^*}{\mu^*} + \frac{4}{3}}. \tag{7}$$

This formula can be used for post-stack PS-wave data, if considering only the vertical quality factor ratio Q_P/Q_S (Arnim and Stewart 2004, Waters 1978, Udias 1999).

The Bulk modulus κ^* is an internal friction loss caused by the compressive stress, while the shear modulus μ^* is caused by the shear stress (Li 1994). Most sedimentary rocks have a low V_P and weak agglutination between particles, and the internal friction represents mainly that the compressive friction is larger than the shear friction, i.e. existing a relationship of $\kappa^* > \frac{4}{3}\mu^*$. When the sedimentary rock has strong agglutination, V_P increases and internal friction is mainly constituted by the shear friction along crevices or inter-granular crevices due to the concretion between particles, and κ^* decreases as μ^* increases, with a relation of $\kappa^* < \frac{4}{3}\mu^*$.

The example seismic data set shown in this paper is from an area covered with Cenozoic sedimentary formations, in huge thickness, un-consolidation and extensively horizontal stratifications and crevices. In that case, the S-wave displays much more attenuation than the P-wave, and $\kappa^* < \frac{4}{3}\mu^*$ holds true. So equation (7) can be expanded to

$$\frac{1}{Q_S} = \frac{3\gamma^2}{4}\left(1 - \frac{3k^*}{4\mu^*} + \left(\frac{3k^*}{4\mu^*}\right)^2 + \cdots\right)\frac{1}{Q_P} \approx \frac{3\gamma^2}{4}\frac{1}{Q_P}. \tag{8}$$

Taking the zeroth-order approximation and substituting it into equation (4) gives

$$\frac{1}{Q_{PS}} \approx \frac{1+\frac{3}{4}\gamma^3}{1+\gamma}\frac{1}{Q_P}. \tag{9}$$

That is, the PS-wave quality factor can be related only to the P-wave quality factor and velocity ratio.

According to equation (9), if $\gamma > 2/\sqrt{3}$, Q_{PS}^{-1} is larger than Q_P^{-1}. This is always the case for seismic media, and since Poisson's ratio is usually larger than 0,γ is always larger than $\sqrt{2}$ (Aki and Richards 1980).

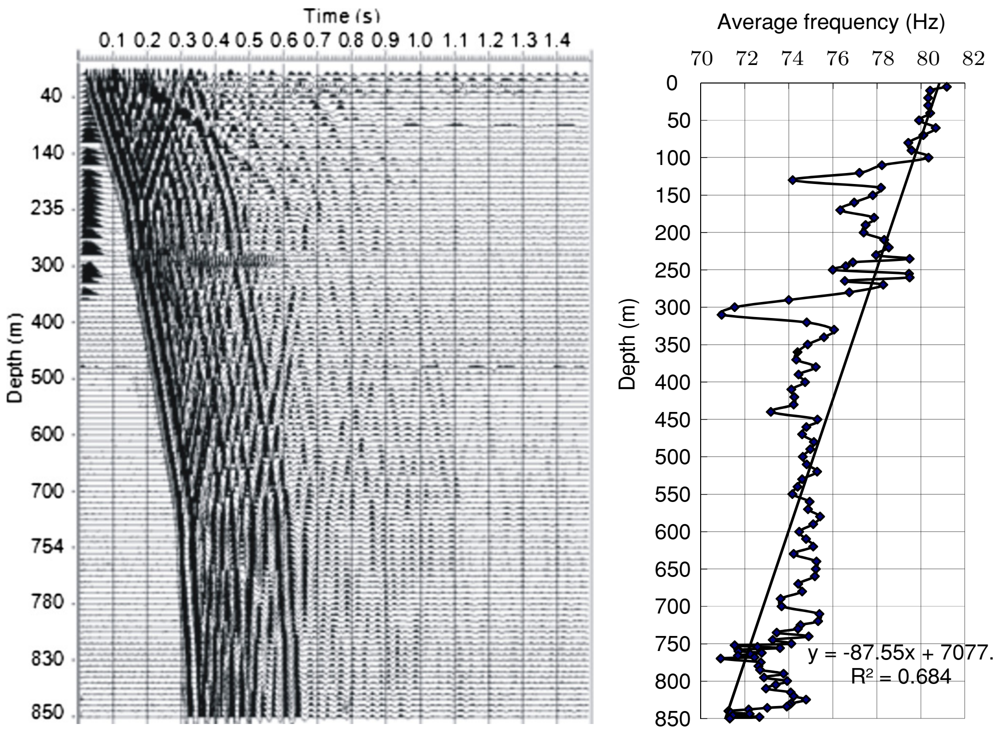
In order to avoid the influence of the inaccuracy of velocity on the quality factor Q_{PS}^{-1}, the velocity ratio γ is computed based on two-way traveltimes of PP and PS events, t_{PP} and t_{PS} respectively. For a zero-offset seismic trace, $V_P = 2D/t_{PP}$, where D is the depth of a reflector. From equation (3), we have $t_{PS} = \frac{1}{2}(t_{PP} - 2D/V_S)$ and then $V_S = 2D/(2t_{PS} - t_{PP})$. Therefore, we obtain

$$\gamma = \frac{2t_{PS} - t_{PP}}{t_{PP}}. \tag{10}$$

As in the derivation above, an accurate estimation of the P-wave quality factor is important for PS-wave Q estimation. The estimation of Q_P is a mature procedure in the seismic data processing and can be found in many references (e.g. Wang 2008a and b).

3. An application example

A VSP data set and two survey lines of two-dimensional three-component (2D3C) seismic data were acquired in a coal-mining field (figure 1(*a*)). The exploration interest is the coal resources between depths of 700 m and 850 m. Near-surface formations with lower density and shallow water table, mainly quaternary sedimentary with thickness from 300 m to



(*a*) (*b*)

Figure 1. (*a*) VSP profile used for the Q estimation. (*b*) The average frequency attenuation versus depth. The straight-line fitting is $y = 7077 - 87.55x$, where x is the frequency and y is the depth. The correlate coefficient (R^2) is 0.684.

Table 1. Estimated P-wave Q values and their lithological descriptions.

Depth (m)	Drilling lithological descriptions	V_P (m s^{-1})	V_P/V_S	Q_P	Q_{PS}
0–70	Superficial loose sandy clay, clay and fine sand formation	1391	4.62	5	0.4
70–180	Coarse sand, sandy clay, gravel, clay	1846	3.98	34	3.5
180–255	Coarse sand, sandy clay, gravel, clay	1903	3.76	36	4.2
255–320	Coarse quartz-sandstone with vertical fractures, efflorescent mudstone	2793	3.37	4	0.6
320–450	Siltstone, mudstone, medium-coarse sandstone, 24th coal seam (1.2 m thickness)	3373	2.35	14	4.4
450–560	Siltstone, mudstone, sandy mudstone, coarse sandstone	3575	1.9	59	27.8
560–710	Interbeded sandstone and mudstone, 17–2nd coal seam (1.5 m thickness), medium sandstone, mudstone	3550	1.89	45	21.4
710–780	Sandy shale, 13–1st coal seam (5.5 m thickness, interesting coal seam), argillaceous siltstone, mudstone	3628	1.87	9	4.4
780–815	Mudstone, sandy mudstone, quartz-sandstone	3961	1.84	14	7
815–850	Sandy mudstone, quartz-sandstone, 11–2nd coal seam (3.5 m thickness, interesting coal seam)	3961	1.84	5	2.5

500 m, are composed of interbedded clayey sand, shale and sand. These geological characters attenuate severely the high-frequency surface seismic components, resulting in narrow bandwidths in deep reflections of PS-wave recorded in surface. Although the continuity of the reflection events is good and the reflection energy of coal seams is also strong enough to be recognized, small structures, especially small faults when the distance of discontinuity less than 5 m, cannot be accurately identified. So, the estimation and compensation of PS-wave Q are necessary for improvement of the resolution of surface seismic data.

The P-wave first arrival has lost 10 Hz high frequency (figure 1(*b*)) due to viscosity from the surface to the depth of 850 m. We use a spectral ratio method (Tonn 1991) to estimate Q_P according to the spectra of VSP first arrivals with the following formula:

$$\frac{|A(\omega)_{d2}|}{|A(\omega)_{d1}|} = \mathrm{e}^{-\frac{|\omega|}{2Q}\left(\frac{d_2}{V_2} - \frac{d_1}{V_1}\right)}, \tag{11}$$

where $A(\omega)_{d_i} (i = 1, 2)$ are the amplitude spectra at depth $d_i (i = 1, 2)$, $\omega = 2\pi f$, V_1 and V_2 are the average velocities at the depth d_1 and d_2 respectively.

According to the interpretation of the VSP data, velocity ratios on each horizon can be derived, and so Q_{PS} values can be calculated with formula (9), as illustrated in table 1.

Figure 2 displays the P-wave velocities, V_P/V_S velocity ratios, Q_P values derived from the VSP first arrivals and

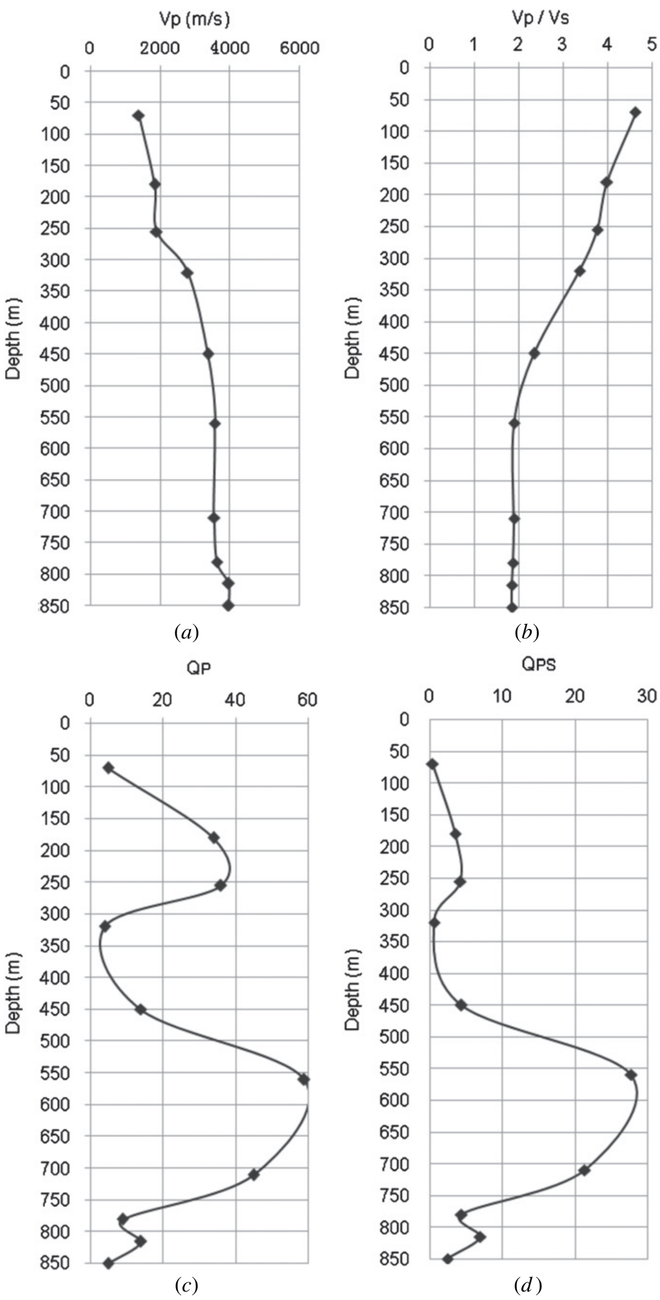


Figure 2. (*a*) V_P versus depth. (*b*) V_P/V_S versus depth. (*c*) Q_P versus depth. (*d*) Q_{PS} versus depth.

Q_{PS} values estimated by equation (9), respectively. From figures 2(*a*) and (*b*), we can see that V_P increases and V_P/V_S decreases as the depth increases. Comparing the estimated Q_P and Q_{PS} values (figures 2(*c*) and (*d*)) with lithological description revealed from drilling information, we can see that the Cenozoic cover, Cenozoic bottom and coal-bearing formations have relatively lower quality factors (the near-surface quaternary soft sedimentary, the unconformity efflorescent surface at the bottom of the Cenozoic and lower density coal seams). We also see that sand-shale formations have relatively higher quality factors responding to the Cenozoic sedimentary and cover of the coal-bearing formations.

4. Conclusions

Through the theoretical analysis of the quality factor in visco-elastic media and field data testing, under the assumption of isotropic media, this short note bears out that the PS-wave frequency is lower than the P-wave evidently, due to viscosity of the real media. By introducing the PS-wave equivalent quality factor, Q estimation of the converted wave data can be simply implemented using the P-wave quality factor and the velocity ratio approximated by the time difference between PP- and PS-events.

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References

Aki K and Richards P G 1980 *Quantitative Seismology* (New York: W H Freeman)

Arnim B H and Stewart R R 2004 Attenuation estimates from VSP and log data *SEG Int'l Exposition and 74th Annual Meeting*

Dasios A, Astin T R and McCann C 2001 Compressional-wave Q estimation from full-waveform sonic data *Geophys. Prospect.* **49** 353–73

Deffenbaugh M 2000 Resolution of converted waves in attenuating media *Expanded Abstracts of 70th SEG Mtg* pp 476–8

Li Q Z 1994 *A Road to Precise Exploration: Engineering Analysis of High Resolution Seismic Exploration Systems* (Beijing: Petroleum Industry Press)

Meissner R 1986 *The Continental Crust—A Geophysical Approach* (Orlando: Academic)

Richard A B and Stewart R R 2002 The impact of attenuation on the resolution of multicomponent seismic data *CREWES Research Report 14* pp 1–21

Tonn R 1991 The determination of seismic quality factor Q from VSP data: A comparison of different computational methods *Geophys. Prosp.* **39** 1–27

Udias A 1999 *Principles of Seismology* (Cambridge: Cambridge University Press)

Wang Y 2002 A stable and efficient approach to inverse Q filtering *Geophysics* **67** 657–63

Wang Y 2003 Quantifying the effectiveness of stabilized inverse Q filtering *Geophysics* **68** 337–45

Wang Y 2004 Q analysis on reflection seismic data *Geophys. Res. Lett.* **31** L17606

Wang Y 2006 Inverse Q-filter for seismic resolution enhancement *Geophysics* **71** V51–60

Wang Y 2008a *Seismic Inverse Q Filtering* (Oxford: Blackwell Publishing)

Wang Y 2008b Inverse-Q filtered migration *Geophysics* **73** S1–6

Wang Y and Guo J 2004 Seismic migration with inverse Q filtering *Geophys. Res. Lett.* **31** L21608

Waters K H 1978 *Reflection Seismology* (New York: Wiley)

White R E 1992 The accuracy of estimating Q from seismic data *Geophysics* **57** 1508–11

Yan H and Liu Y 2009 Estimation of Q and inverse Q filtering for prestack reflected PP- and converted PS-waves *Appl. Geophys.* **6** 59–69

Zhao S and Wang Y 2004 Relative amplitude preserved PP and PSV wave seismic sections *J. Geophys. Eng.* **1** 259–62