Random nosie contained seismic data interpolation via thresholding method

Jingjie Cao, Shijiazhuang University of Economics, China*
Yanfei Wang, Institute of Geology and Geophysics, CAS, China

Summary

Sparsity-promoted seismic interpolation method is the mainstream of interpolation methods. However, most sparsity-promoted methods are target at high signal-to-noise ratio data and can not handle noise contained data. The existence of noise makes the interpolation results unacceptable, thus simultaneous interpolation and denoising is necessary to remove noise and interpolate data. This abstract demonstrate that random noise contained seismic data interpolation can be realized by adjusting the least thresholds of thresholding methods. Furthermore, the weighted projection onto convex sets method was proved to be a thresholding method aimming at a unconstrained optimization model but the regularization parameters are amplified by the weighting factor. Additionally, a novel thresholding method based on zero-norm regularization is introduced for simultaneous interpolation and denoising. Numerical examples on synthetic and field data demonstrate that the proposed thresholding method can get reliable results for random noise contained data interpolation with high efficiency.

Introduction

In seismic exploration, field data often violates the Shannon/Nyquist sampling theorem due to influences of acquisition costs, bad traces, topography and noise. The incomplete data may affect results of 3D surface-related multiple elimination (Berkhout and Verschuur, 1997), wave-equation pre-stack depth migration (Claerbout, 1971) and time-lapse imaging (Smith et. al., 2012), thus interpolation must be adopted to provide complete wavefield information. Among various interpolation methods, prediction filter methods (Spitz, 1991) are mainly target at regular sub-sampled data; wave-equation based methods (Ronen, 1987; Stolt, 2002) call for underground parameters and are time consuming; matrix/tensor completion methods (Oropeza and Sacchi, 2011; Ma, 2013) assume that seismic data can be restored by minimization of the rank of matrix/tensor. Currently, sparse transform based methods are the mainstream of interpolation, Fourier transform (Liu and Sacchi, 2004; Xu et. al., 2005), Radon transform (Herrmann et. al., 2000) and curvelet transform (Herrmann and Hennenfent, 2008; Wang et. al., 2011; Cao et. al., 2012) were chosen as the sparse transform to express seismic data sparsely in their individual domains. Actually, field seismic data inevitably contain different kinds of noises, especially the random white noise. During the above mentioned methods, few methods aim at random noise contained seismic data interpolation. Oropeza and Sacchi (2011) proposed a weighted reinsert method for simultaneous interpolation and denoising, where the random white noise can be removed by adjusting a weighting factor. Similarly, Gao et. al. (2013) proposed a weighted projection onto convex sets method (POCS) to realize denoising and interpolation.

This abstract analyzed the mathematical model of interpolation and claim that it is also suitable for random noise contained data interpolation, the weighted POCS method was proved to be a threshoding method for unconstrained optimization model. A novel thresholding method is also proposed to realize simultaneous interpolation and denoising; the curvelet transform is adopted as the sparse transform. Numerical examples on synthetic and field seismic data demonstrate that the proposed thresholding method can get reliable results for random noise contained data interpolation.

Theory

Sparse transform based seismic interpolation can be implemented by solving the following problem:

$$\min \frac{1}{2}\left\|\Phi\Psi x - d_{obs}\right\|_2^2 + \lambda\left\|x\right\|_0 , \tag{1}$$

where Φ is the sampling matrix, d_{obs} is the observed data, x is the sparse expression of seismic signal in certain transformed domain, Ψ is a sparse transform, and λ is the regularization parameter to balance the fitting error and sparsity of x , $\left\|x\right\|_0$ denots the non-zero elements number of x . Most interpolation methods for solving problem (1) can only solve noiseless or high quality data; however, problem (1) is also suitable for random noise contained data interpolation. The key point of using problem (1) for simultaneous interpolation and denoising is choosing a proper λ . Generally, small λ corresponds to weak noise, and large λ corresponds to strong noise. Essentially, thresholding in a transform domain is a denoising process, λ has some relationship to the energy of noise, the best balance between accuracy of the solution and denoising can be obtained when choosing a proper λ . Because the noise energy is different from data to data, λ should be given according to experience.

The POCS method is a commonly used method for interpolation, however, it can not handle noise contained data interpolatin. The weighted POCS method was proposed in Gao et. al., 2013 to realize noise contained data

DOI http://dx.doi.org/10.1190/segam2015-5796261.1

interpolation. A weighting factor was induced during the projection step of the original POCS method. In the following, we prove that the weighted POCS method is essentially a threshold method for problem (1) but the regularization parameter λ was amplified by the weighting factor. Defining $A = \Phi\Psi$ and $f(x) = \frac{\alpha}{2}\|Ax - d_{obs}\|_2^2 + \lambda\|x\|_0$, the iteration scheme of the thresholding method (Yang et. Al., 2012) for solving $\min f(x)$ is

$$\begin{aligned} x^{k+1} &= H_{\tau_k}\left(x^k - \nabla f\left(x^k\right)\right) \\ &= H_{\tau_k}\left(x^k - \alpha A^T\left(Ax - d_{obs}\right)\right), \qquad (2) \\ k &= 1,2,\cdots,N \end{aligned}$$

where τ_k is the kth thresholding parameter, N is the iterative number, $H_{\tau_k}(\bullet)$ is the hard thresholding operation. If we multiply operator Ψ from the left side of equation (2) , then iteration scheme (2) can be changes to $d^{k+1} = \Psi x^{k+1} = \Psi H_{\tau_k}\left(x^k + \alpha(\Phi\Psi)^T\left(d_{obs} - \Phi d^k\right)\right)$.

Since $\Phi^T = \Phi = (\Phi)^2$ and $\Phi d_{obs} = \Phi^2 d = d_{obs}$, the above equation can be transformed to

$$\begin{aligned} d^{k+1} &= \Psi H_{\tau_k}\left(x^k + \alpha(\Phi\Psi)^T\left(d_{obs} - \Phi d^k\right)\right) \\ &= \Psi H_{\tau_k}\left(\Psi^T\left(\alpha d_{obs} + (I - \alpha\Phi)d^k\right)\right) \end{aligned}, \qquad (3)$$

where I is the identity matrix when d^k is its vertor form and the all-ones matrix if d^k is treated as a matrix. Defining $u^k = \alpha d_{obs} + (I - \alpha\Phi)d^k$, then equation (3) can be changed to $d^{k+1} = \Psi H_{\tau_k}\left(\Psi^T\left(u^k\right)\right)$. Thus,

$$\begin{aligned} u^{k+1} &= \alpha d_{obs} + (I - \alpha\Phi)d^{k+1} \\ &= \alpha d_{obs} + (I - \alpha\Phi)\Psi H_{\tau_k}\left(\Psi^T\left(u^k\right)\right) \end{aligned}. \qquad (4)$$

This is the iteration scheme of the weighted POCS method (Gao et. al., 2013). According to the above discussion, the weighting factor α is actually the weighting factor in problem (1). Therefore, the weighted POCS method equals to solve

$$\min \frac{1}{2}\|Ax - d_{obs}\|_2^2 + \frac{\lambda}{\alpha}\|x\|_0 . \qquad (5)$$

We must indicate that the weighted POCS method is not a projection onto convex set method when $\alpha \neq 1$. u^k is the corresponding time-space expression of $x^k + \alpha A^T\left(d_{obs} - Ax^k\right)$. When $\alpha = 1$, it changed to $u^k = d_{obs} + (I - \Phi)d^k$, this is the projection of d^k onto convex $S = \{d \mid \Phi d = d_{obs}\}$. However, if $\alpha \neq 1$, u^k will not be a projection process, thus weighted POCS method is actually a thresholding method for problem (1).

Based on the above analysis, we can realize simultaneous interpolation and denoising by solving problem (1) if the regularization parameter is proper. In the next, we introduce a novel thresholding method to solve problem (1) (Elad *et al.* 2005). The algorithm is as follows:

Step 1. Input the sampled data d_{obs}, the sampling matrix Φ, the sparse transform Ψ, the maximum iterative number N, and the least threshold τ, $k = 0$.

Step 2. Let the initial solution $d^0 = 0$, then the residual is $r^0 = d_{obs}$, the initial threshold $\tau_0 = \max\left\|\Psi^T d_{obs}\right\|_\infty$.

Step 3. Calculate the prediction residual $r^{k+1} = r^k + d^k$, update the transform coefficients by hard thresholding operation $\alpha^{k+1} = H_{\tau_k}(\Psi^T r^{k+1})$, and update the iterative solution $d^{k+1} = \Psi\alpha^{k+1}$.

Step 4. Update the residual $r^{k+1} = d_{obs} - \Phi d^{k+1}$, and then reduce the threshold. If $k \geq N$, turn to step 5, otherwise, it turns to Step 3.

Step 5. Output the final solution $d^f = d^N$.

In order to improve convergence of this algorithm, the exponential reduction strategy of the thresholds which has proved to be very fast for interpolation is adopted here (Gao et. al., 2013).

Examples

The interpolation and denoising ability of the new thresholding method is evaluated on the following examples. For the first example, a synthetic data containing additive random noise is used to test the efficiency and accuracy of the proposed thresholding method. The POCS method is used as the benchmark. A synthetic random white noise contained data with SNR= 4.3049 db is shown in Figure 1(a). The time sampling number is 701 with time sampling interval 4ms, the space sampling number is 300 with trace distance 20 m. Figure 1(b) is a randomly sub-sampled version of Figure 1(a) with SNR =1.5004 db. Parameters are the same: the least threshold is 0.00035 and the maximum iterative number is 60. The restoration of the POCS method is shown in Figure 1(c) with SNR= 7.2361 db. Figure 1(d) is the restoration of the proposed thresholding method with SNR= 17.8514 db. The 50th trace of Figure 1(a), Figure 1(d) and the noiseless data are plotted in Figure 2. The restored SNR of both methods are shown in Figure 3. Based on these results, it

DOI http://dx.doi.org/10.1190/segam2015-5796261.1

can be conclude that the proposed thresholding method result in much better results than the POCS method.

Figure 4(a) is a field data with 276 time samples and 232 traces, the time sampling rate is 4 ms and the trace distance is 25 m, some noise is contained in the field data such that the signal-noise-ratio should be improved; Figure 4(b) is the F-K spectrum of Figure 4(a);Figure 4(c) is a randomly sampled version of Figure 4(a) with 50% traces removed. Figure 4(d) is the F-K spectrum of Figure 4(c), Figure 4(e) is the restoration of the new thresholding method, and Figure 4(f) is the F-K spectrum of Figure 4(e). The least threshold is 0.04 and the maximum iteration number is 60. It is obvious that the proposed thresholding method can restore missing traces and remove noise simultaneously. Deoising can be avoid after the proposed thresholding interpolation.

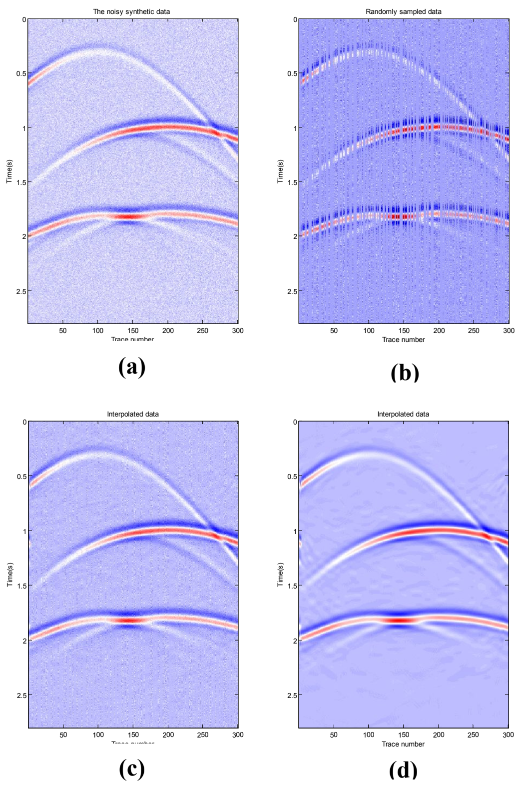

Figure 1: (a) Noisy synthetic data with SNR=4.3049 db; (b) Sampled noisy data with SNR=1.5004 db. (c) Interpolation result of the POCS method with SNR=7.2361db; (d) Interpolation of the proposed thresholding method with SNR=17.8514db.

Conclusions

In this abstract, we claimed that simultaneous interpolation and denoising can be realized by choosing proper least thresholds when thresholding methods were adopted. We also demonstrate that the weighted POCS method is a thresholding method for unconstrained problem (1) but the regularization parameters are amplified by the weighting factor. Additionally, a novel thresholding method is introduced to realize simultaneous interpolation and denoising. Numerical experiments demonstrate that the proposed thresholding method can get acceptable results for random noise contained data interpolation, and denoising can be avoid after interpolation.

Acknowledgments

This work is supported by National Natural Science Foundation of China under grant numbers 41204075, 11271349 and Natural Science Foundation of Hebei Province under grant number D2014403007.

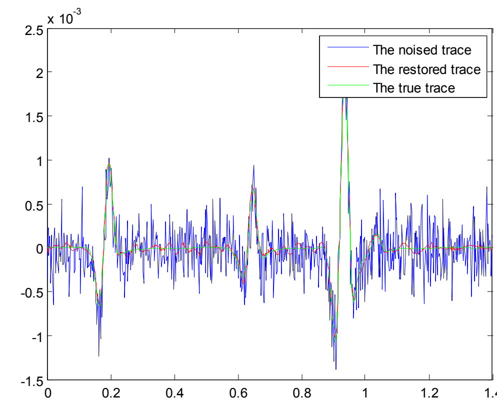

Figure 2: The 50th trace of Figure 1(a), Figure 1(d) and the original noiseless data.

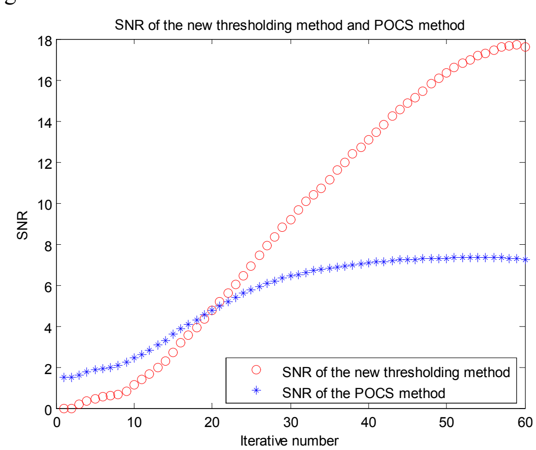

Figure 3: Convergence diagram of noisy synthetic data.

DOI http://dx.doi.org/10.1190/segam2015-5796261.1

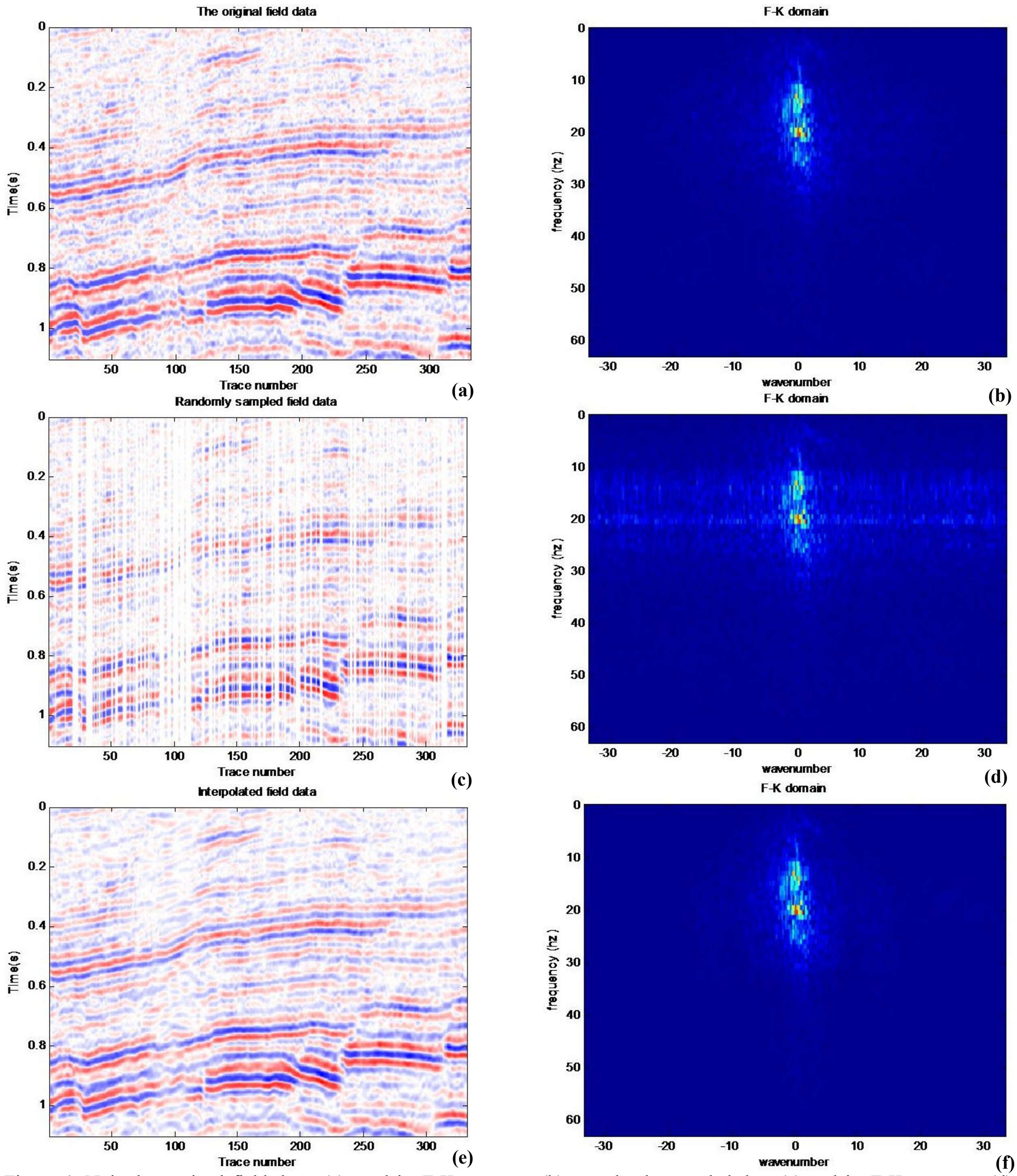

Figure 4: Noised contained field data (a) and its F-K spectrum (b); randomly sampled data (c) and its F-K spectrum (d); Interpolation of the proposed method (e) and its F-K spectrum (f).

DOI http://dx.doi.org/10.1190/segam2015-5796261.1

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2015 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Berkhout, A. J., and D. J. Verschuur, 1997, Estimation of multiple scattering by iterative inversion, Part I: Theoretical considerations: Geophysics, **62**, 1586–1595, http://dx.doi.org/10.1190/1.1444261.

Cao, J. J., Y. F. Wang, and C. C. Yang, 2012, Seismic data restoration based on compressive sensing using regularization and zero-norm sparse optimization: Chinese Journal of Geophysics, **55**, no. 2, 239–251, http://dx.doi.org/10.1002/cjg2.1718 (in Chinese).

Claerbout, J. F., 1971, Toward a unified theory of reflector mapping: Geophysics, **36**, 467–481, http://dx.doi.org/10.1190/1.1440185.

Elad, M., J. L. Starck, P. Querre, and D. Donoho, 2005, Simultaneous cartoon and texture image inpainting using morphological component analysis (MCA): Applied and Computational Harmonic Analysis, **19**, no. 3, 340–358, http://dx.doi.org/10.1016/j.acha.2005.03.005.

Gao, J. J., A. Stanton, M. Naghizadeh, M. D. Sacchi, and X. H. Chen, 2013, Convergence improvement and noise attenuation considerations for beyond alias projection onto convex sets reconstruction: Geophysical Prospecting, **61**, 138–151, http://dx.doi.org/10.1111/j.1365-2478.2012.01103.x.

Hennenfent, G., and F. Herrmann, 2008, Simply denoise: Wavefield reconstruction via jittered undersampling: Geophysics, **73**, no. 3, V19–V28, http://dx.doi.org/10.1190/1.2841038.

Herrmann, F., and G. Hennenfent, 2008, Non-parametric seismic data recovery with curvelet frames: Geophysical Journal International, **173**, no. 1, 233–248, http://dx.doi.org/10.1111/j.1365-246X.2007.03698.x.

Herrmann, P., T. Mojesky, M. Magasan, and P. Hugonnet, 2000. De-aliased, high-resolution Radon transforms: 70th Annual International Meeting, SEG, Expanded Abstracts, 1953–1956.

Liu, B., and M. D. Sacchi, 2004, Minimum weighted norm interpolation of seismic records: Geophysics, **69**, 1560–1568, http://dx.doi.org/10.1190/1.1836829.

Ma, J. W., 2013, Three-dimensional irregular seismic data reconstruction via low-rank matrix completion: Geophysics, **78**, no. 5, V181–V192, http://dx.doi.org/10.1190/geo2012-0465.1.

Oropeza, V., and M. D. Sacchi, 2011, Simultaneous seismic data denoising and reconstruction via multichannel singular spectrum analysis: Geophysics, **76**, no. 3, V25–V32, http://dx.doi.org/10.1190/1.3552706.

Ronen, J., 1987, Wave-equation trace interpolation: Geophysics, **52**, 973–984, http://dx.doi.org/10.1190/1.1442366.

Smith, P. J., I. Scott, and T. Traylen, 2012, Simultaneous time-lapse binning and regularization of 4D data: 74th Conference & Exhibition, EAGE, Extended Abstracts, E009.

Spitz, S., 1991, Seismic trace interpolation in the *f-x* domain: Geophysics, **56**, 785–794, http://dx.doi.org/10.1190/1.1443096.

Stolt, R. H., 2002, Seismic data mapping and reconstruction: Geophysics, **67**, 890–908, http://dx.doi.org/10.1190/1.1484532.

Wang, Y. F., J. J. Cao, and C. C. Yang, 2011, Recovery of seismic wavefields based on compressive sensing by an *l*1-norm constrained trust region method and the piecewise random subsampling: Geophysical Journal International, **187**, no. 1, 199–213, http://dx.doi.org/10.1111/j.1365-246X.2011.05130.x.

Xu, S., Y. Zhang, D. Pham, and G. Lambaré, 2005, Antileakage Fourier transform for seismic data regularization: Geophysics, **70**, no. 4, V87–V95, http://dx.doi.org/10.1190/1.1993713.

Yang, P. L., J. H. Gao, and W. C. Chen, 2012, Curvelet-based POCS interpolation of non-uniformly sampled seismic records: Journal of Applied Geophysics, **79**, 90–99, http://dx.doi.org/10.1016/j.jappgeo.2011.12.004.

DOI http://dx.doi.org/10.1190/segam2015-5796261.1